The following list of formulae may be found useful :

Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$

Newton's law of gravitation $F = \frac{G m_1 m_2}{r^2}$

Use the following data wherever necessary :

Acceleration due to gravity $g = 9.81$ m s^{-2} (close to the Earth)

Universal gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Part A : HKAL examination questions

1. **< HKAL 1980 Paper I - 2 >**

Two identical spheres, each of mass M and radius r are in contact. One sphere is displaced by a distance $4r$, along the line of centres, away from the first sphere. What is the ratio of the final gravitational force between the sphere to the initial gravitational force between them ?

A. 1 : 3
B. 1 : 9
C. 1 : 16
D. 1 : 25

2. **< HKAL 1984 Paper I - 40 >**

Taking the Earth to be a perfect sphere with uniform density, which of the following statements concerning the gravitational field g of the Earth is/are correct ?

(1) The gravitational field at the surface of the Earth is greater than that at the top of a high mountain.
(2) If the density of the Earth increases with its radius remaining unchanged, g at the surface increases.
(3) If the radius of the Earth increases with its density remaining unchanged, g at the surface decreases.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

3. **< HKAL 1991 Paper I - 1 >**

In which of the following situations is the magnitude of the normal reaction R of the supporting surface equal to the weight mg of the body ?

(1) A body is resting on a rough inclined plane.
(2) A body placed on the floor inside a spacecraft in circular orbit around the Earth.
(3) A body placed on the floor of a lift moving upwards with uniform velocity.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

4. **< HKAL 1993 Paper I - 12 >**

A parking satellite appears stationary vertically above an observer at the equator of the Earth. The radius of the satellite from the Earth is 4.24×10^7 m. Calculate the mass of the Earth.

A. 4.5×10^{24} kg
B. 5.0×10^{24} kg
C. 5.5×10^{24} kg
D. 6.0×10^{24} kg

5. **< HKAL 1996 Paper IIA - 2 >**

In which of the following cases would the resultant force on the object become zero ?

(1) a satellite moving round the Earth
(2) a feather falling freely in a vacuum cylinder in a laboratory
(3) a parachutist falling with terminal velocity in air

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

6. **< HKAL 1997 Paper IIA - 10 >**

A close-orbit satellite near the Earth's surface has a speed of 7900 m s^{-1}. The radius of the Earth is 4 times that of the Moon and the ratio of the average density of the Earth to that of the Moon is 5 : 4. What would be the speed of a close-orbit satellite near the Moon's surface ?

A. 1770 m s^{-1}
B. 2210 m s^{-1}
C. 2470 m s^{-1}
D. 3570 m s^{-1}

7. **< HKAL 2000 Paper IIA - 9 >**

There are two planets X and Y. Each of them has a close-orbit satellite revolving close to the planet. If the two satellites are observed to have the same period, then X and Y must have nearly the same

A. mass.
B. average density.
C. radius.
D. acceleration due to gravity at the planet's surface.

8. **< HKAL 2000 Paper IIA - 2 >**

In the following situations, which of the cases would the normal reaction acting on a body and the weight of the body have the same magnitude ?

(1) A ball bouncing vertically on a horizontal ground is in contact with the ground.
(2) An astronaut in a spacecraft which performs circular motion around the Earth.
(3) A boy standing in a lift which is moving vertically upward with a uniform velocity.

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

9. **< HKAL 2003 Paper IIA - 11 >**

Assume that the Earth is a perfect sphere. If the radius of the Earth is 6.4×10^6 m, what is its average density ?

A. 5.5×10^3 kg m^{-3}
B. 7.3×10^3 kg m^{-3}
C. 2.3×10^4 kg m^{-3}
D. 6.0×10^{24} kg m^{-3}

10. **< HKAL 2004 Paper IIA - 8 >**

There are many satellites revolving around the Saturn. Different satellites have different speed v and radius r. Which of the following correctly express the relation between these two values ?

A. $v \propto r$
B. $v \propto \sqrt{r}$
C. $v \propto \frac{1}{r}$
D. $v \propto \frac{1}{\sqrt{r}}$

11. **< HKAL 2004 Paper IIA - 7 >**

A planet has a mass 3 times that of the Earth and a diameter 2 times that of the Earth. What is the gravitational field strength on the planet's surface ?

A. 7.36
B. 9.81
C. 14.7
D. 19.6

12. **< HKAL 2005 Paper IIA - 28 >**

An object of mass 25 kg has a weight of 41 N on the surface of the moon. The radius of the moon is R. What is the gravitational field strength in N kg^{-1}, at a point distance $2R$ from the centre of the moon ?

A. 1.64
B. 0.82
C. 0.41
D. 0.21

13. **< HKAL 2006 Paper IIA - 28 >**

The Earth is at a distance r from the centre of the Sun. It takes 365 days for the Earth to revolve once around the sun in a circular path. Find the mass of the Sun in terms of r.

A. $2.45 \times 10^{-4} r^3$
B. $5.95 \times 10^{-4} r^3$
C. $3.85 \times 10^{4} r^3$
D. $1.75 \times 10^{6} r^3$

14. **< HKAL 2007 Paper IIA - 5 >**

In which of the following situations does the person concerned experience 'weightlessness' ?

(1) an astronaut in a spacecraft which is decelerating to make a soft landing on the moon
(2) a parachutist descending with a constant velocity in the air
(3) an astronaut in a spacecraft which is orbiting around the Earth with its rocket engines shut off

A. (1) only
B. (3) only
C. (1) & (2) only
D. (2) & (3) only

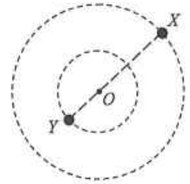
15. **< HKAL 2008 Paper IIA - 27 >**

Which of the following statements about parking orbits around the Earth are correct ?

(1) All satellites in a parking orbit must have the same speed.
(2) No satellite in a parking orbit can pass vertically above Hong Kong.
(3) There is only one parking orbit around the Earth.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

16. **< HKAL 2008 Paper IIA - 28 >**



The above figure shows a binary star system in which X and Y are two stars revolving about O with uniform circular motion under their mutual gravitational attraction. If the radius of the orbit of X is twice that of Y, which of the following deductions are correct ?

(1) The acceleration of X is twice that of Y.
(2) The orbital speed of X is equal to that of Y.
(3) The mass of X is half that of Y.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

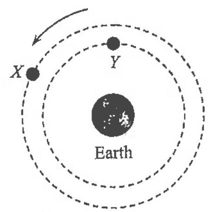
17. **< HKAL 2010 Paper IIA - 12 >**

Ganymede is one of the satellites of Jupiter. The radius of Ganymede's orbit around Jupiter is about 3 times that of the Moon around the Earth. The mass of Jupiter is 318 times that of the Earth. If the period of the Moon around the Earth is 27.3 days, what is the period of Ganymede revolving around Jupiter ?

A. 2.7 days
B. 8.0 days
C. 91 days
D. 273 days

Part B : Supplemental exercise

18. Two identical satellites X and Y are moving in two circular orbits around the Earth as shown. Which statement is/are correct ?



(1) The period of X is greater than that of Y.
(2) The speed of X is smaller than that of Y.
(3) The gravitational force on X is smaller than that on Y.

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

19. Given that the radius of the Earth is 6380 km. Find the acceleration due to gravity at a height of 3200 km.

A. 3.65 N kg^{-1}
B. 4.35 N kg^{-1}
C. 5.85 N kg^{-1}
D. 6.75 N kg^{-1}

20. The radius of the Earth is R. Satellite X orbits around the Earth at a height of R, while satellite Y orbits around the Earth at a height of 2 R. Find the ratio of the speed of X to that of Y.

A. $\sqrt{\frac{1}{2}}$

B. $\sqrt{\frac{2}{1}}$

C. $\sqrt{\frac{2}{3}}$

D. $\sqrt{\frac{3}{2}}$

21. On a certain planet, an object is thrown vertically upwards with an initial velocity of v_1 and it returns to the ground after time t. If v_2 is the orbital speed of a satellite circling close to the planet, what is the radius of the planet ?

A. $\frac{2v_1^2 t}{v_2}$

B. $\frac{4v_1^2 t}{v_2}$

C. $\frac{2v_2^2 t}{v_1}$

D. $\frac{v_2^2 t}{2v_1}$

22. If the gravitational constant G becomes larger while the orbital radius of the Moon around the Earth and the masses of the Moon and the Earth remain unchanged, which physical quantity of the Moon would change ?

(1) orbital speed
(2) period revolving around the Earth
(3) acceleration

A. (1) & (2) only
B. (1) & (3) only
C. (2) & (3) only
D. (1), (2) & (3)

23. A small sphere X of mass M is placed at a distance d from a point mass. The gravitational force the sphere X is 120 N. The sphere X is removed and another sphere Y of mass $3M$ is placed at a distance $2d$ from the same point mass. What would then be the gravitational force on the sphere Y?

A. 80 N
B. 90 N
C. 160 N
D. 180 N

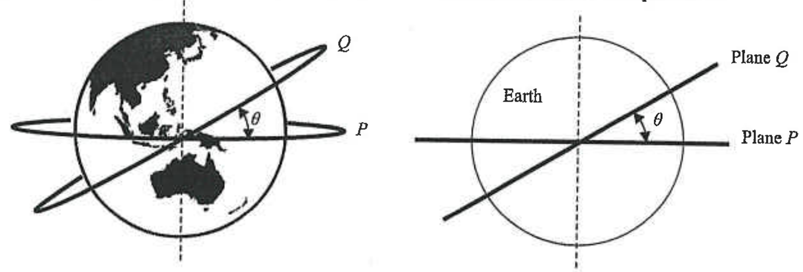
24. A parking satellite is moving at a constant speed in a circular orbit around the Earth. At any instant, the resultant force acting on the satellite is

A. zero.
B. equal to the gravitational force on the satellite.
C. equal to the resultant force of the gravitational force on the satellite and the centripetal force.
D. equal to the force exerted by the rockets of the satellite.

Part C : HKDSE examination questions

25. < HKDSE 2012 Paper IA - 14 >

Two satellites move in circular orbits of the same radius R around the Earth (mass M). The orbits are in two different planes P and Q as shown. Plane P coincides with the Earth's equator while plane Q is inclined to the equator at θ.



Which of the following statement is INCORRECT ?

A. The speed of satellite P is $\sqrt{\frac{GM}{R}}$.
B. The centripetal force acting on satellite Q is pointing along the plane Q.
C. The acceleration of both satellites is the same in magnitude.
D. The period of satellite Q is longer than that of satellite P.

26. < HKDSE 2013 Paper IA - 15 >

It is known that the mass of Mars is about $\frac{1}{10}$ of that of the Earth while its radius is about $\frac{1}{2}$ of the Earth's radius. In terms of the gravitational acceleration g on the Earth's surface, the approximate gravitational acceleration on the surface of Mars is

A. 0.2 g
B. 0.4 g
C. 2.5 g
D. 4 g

27. < HKDSE 2014 Paper IA - 11 >

An astronaut inside a spacecraft moving in a circular orbit around the Earth is apparently weightless because

A. the astronaut is too far from the Earth to feel the Earth's gravitational force.
B. the astronaut and the spacecraft are both moving with the same acceleration due to gravity towards the Earth.
C. the Earth's gravitational force on the astronaut is balanced by the reaction force of the spacecraft's floor.
D. the Earth's gravitational force on the astronaut is balanced by the centripetal force.

28. < HKDSE 2014 Paper IA - 12 >

An artificial satellite revolves in a circular orbit above the Earth's surface at a height equal to the radius of the Earth. Find the acceleration of the satellite in terms of the acceleration due to gravity g on the Earth's surface.

A. $\frac{1}{8}g$
B. $\frac{1}{4}g$
C. $\frac{1}{2}g$
D. g

29. **< HKDSE 2015 Paper IA - 11 >**

The gravitational force exerted on the Earth by the Sun is F_o. The gravitational force exerted on the Sun by the Earth is

A. equal to F_o and in the same direction.
B. equal to F_o and in the opposite direction.
C. much smaller than F_o and in the same direction.
D. much smaller than F_o and in the opposite direction.

30. **< HKDSE 2016 Paper IA - 14 >**

A satellite orbits the Earth in a circular path of radius 7.2×10^6 m. What is the period of the satellite ?

Given : mass of the Earth = 6.0×10^{24} kg

A. 1.4 hours
B. 1.7 hours
C. 1 day
D. Answer cannot be found as the mass of the satellite is not known.

31. **< HKDSE 2017 Paper IA - 13 >**

A small object is released from rest at a point very far away from a planet X. The object then starts moving towards X. X does not have an atmosphere. Neglect the effect of other celestial bodies.

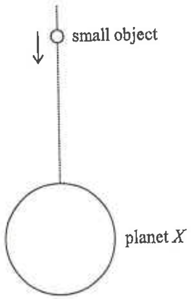
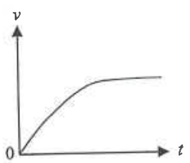


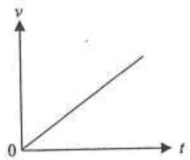
Diagram **NOT** shown to scale

Which of the following graphs best shows the variation of the velocity v of the object with time t before it hits X ?

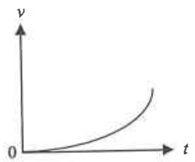
A.



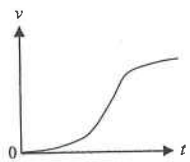
B.



C.



D.



32. **< HKDSE 2018 Paper IA - 13 >**

A satellite of mass m moves around a planet of mass M in circular orbit of radius r. What does the angular velocity of the satellite depend on ?

(1) r
(2) m
(3) M

A. (1) only
B. (2) only
C. (1) & (3) only
D. (2) & (3) only

33. <HKDSE 2020 Paper IA-10>

The diameter of Neptune is about 4 times that of the Earth and its mass is about 17 times that of the Earth. Estimate the acceleration due to gravity on Neptune's surface.
Given: acceleration due to gravity on Earth's surface g = 9.81 m s^{-2}

A. 2.3 m s^{-2}
B. 9.2 m s^{-2}
C. 10.4 m s^{-2}
D. 41.7 m s^{-2}

HKEAA's Marking Scheme is prepared for the markers' reference. It should not be regarded as a set of model answers. Students and teachers who are not involved in the marking process are advised to interpret the Marking Scheme with care.

M.C. Answers

1. B	11. A	21. D	31. C
2. C	12. C	22. D	32. C
3. B	13. B	23. B	33. C
4. D	14. B	24. B	
5. B	15. D	25. D	
6. A	16. B	26. B	
7. B	17. B	27. B	
8. B	18. D	28. B	
9. A	19. B	29. B	
10. D	20. D	30. B	

M.C. Solution

1. B

$$F = \frac{GMM}{r^2} \propto \frac{1}{r^2}$$

$$\frac{F_2}{F_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{r+r}{r+4r+r}\right)^2 = \frac{1}{9}$$

2. C

✓ (1) By $g \propto \frac{1}{r^2}$. At the top of a high mountain, r is greater, thus g is smaller.

✓ (2) Let ρ be the density of the Earth and V be the volume of the Earth.
By $g = \frac{GM}{R^2} \propto \frac{\rho V}{R^2} \propto \frac{\rho V}{R^2} \propto \frac{\rho R^3}{R^2} \propto \rho R$
As radius R is unchanged, $g \propto \rho$, thus g increases if density ρ increases.

× (3) As density ρ is unchanged, $g \propto R$, thus g should increase if radius R increases.

3. B

× (1) Resting on a rough inclined plane : $R = mg\cos\theta \neq mg$

× (2) Since the weight is completely used to provide the centripetal force, the body is not in contact with the floor, $R = 0$.

✓ (3) Moving upwards with uniform velocity means no acceleration, thus $R = mg$.

4. D

$$\frac{GMm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$(6.67\times10^{-11})M = (4.24\times10^7)^3\left(\frac{2\pi}{24\times60\times60}\right)^2$$

$\therefore$ $M = 6.0 \times 10^{24}$ kg

5. B

× (1) A satellite moving round the earth is in circular motion, there must be a resultant force towards the centre (centripetal force) acting on the satellite.

× (2) The resultant force of a feather falling freely in vacuum is the gravitational attraction force (the weight).

✓ (3) For any object moving in terminal velocity (uniform velocity), acceleration = 0, thus, net force = 0

6. A

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{GM}{R}} \propto \sqrt{\frac{\rho \cdot V}{R}} \propto \sqrt{\frac{\rho \cdot R^3}{R}} \propto \sqrt{\rho \cdot R^2} \propto \sqrt{\rho} \cdot R$$

$$\frac{v_m}{v_e} = \sqrt{\frac{\rho_m}{\rho_e}} \cdot \frac{R_m}{R_e}$$

$\therefore$ $\frac{v_m}{7900} = \sqrt{\frac{4}{5}} \cdot \frac{1}{4}$ $\therefore$ $v_m = 1770$ m s^{-1}

7. B

$$\frac{GMm}{R^2} = mR\omega^2$$

$\therefore$ $\frac{GM}{R^3} = \omega^2 = \left(\frac{2\pi}{T}\right)^2$

$\therefore$ Same $T \Rightarrow$ same $\frac{M}{R^3} \Rightarrow$ same $\frac{M}{V} \Rightarrow$ same ρ

(Volume : $V = \frac{4}{3}\pi R^3 \propto R^3$) (density : $\rho = \frac{M}{V}$)

8. B

× (1) When the bouncing ball is in contact with the ground, normal reaction R is given by $R - mg = (mv - mu)/t$ $\therefore$ $R > mg$.

× (2) Since the weight is completely used to provide the centripetal force in circular motion, the body is not in contact with the floor, $R = 0$.

✓ (3) Moving upwards with uniform velocity means no acceleration, thus $R = mg$.

9. A

At the surface of the Earth,

$$g = \frac{GM}{R^2}$$

$$\therefore (9.81) = \frac{(6.67\times10^{-11})M}{(6.4\times10^6)^2}$$

$\therefore M = 6.024 \times 10^{24}$ kg

Average density of the Earth :

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{(6.024\times10^{24})}{\frac{4}{3}\pi(6.4\times10^6)^3} = 5500 \text{ kg m}^{-3}$$

10. D

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore v^2 = \frac{GM}{r}$$

$$\therefore v \propto \frac{1}{\sqrt{r}}$$

11. A

$$g = \frac{GM}{R^2} \propto \frac{M}{d^2}$$

$$\therefore \frac{g_P}{g_E} = \left(\frac{M_P}{M_E}\right)\cdot\left(\frac{d_E}{d_P}\right)^2 = (3)\times(\frac{1}{2})^2 = \frac{3}{4}$$

$$\therefore g_P = \frac{3}{4}\times 9.81 = 7.36$$

12. C

① $W = mg$ $\therefore (41) = (25)g$ $\therefore g = 1.64$ N kg^{-1}

② $g \propto \frac{1}{r^2}$ $\therefore g' = 1.64 \times (\frac{R}{2R})^2 = 0.41$ N kg^{-1}

13. B

By $\frac{GMm}{r^2} = mr\omega^2$

$$\therefore \frac{GM}{r^3} = \omega^2 = (\frac{2\pi}{T})^2$$

$$\therefore M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 r^3}{(6.67\times10^{-11})\cdot(365\times24\times3600)^2} = 5.95\times10^{-4}\, r^3$$

14. B

× (1) When the spacecraft decelerates downwards, normal reaction acts on the astronaut so that $R - mg = ma$, the astronaut feels heavier.

× (2) The air resistance that is equal to the weight acts on the parachutist to give him the feeling of weight.

✓ (3) When the rocket engines are shut off, the weight of the astronaut is used completely to provide his centripetal acceleration. As no normal reaction acts on him, he experiences weightlessness.

15. D

✓ (1) Speed of satellites depends on the radius of the orbit, as the radius of parking orbit is fixed, the speed is also a specified value.

✓ (2) Hong Kong is not at the equator of the Earth, thus there is no parking satellite directly above HK.

✓ (3) Parking orbit must be at the equatorial plane and at a certain height above the Earth surface, thus there is only one orbit. Note that parking orbit is geostationary orbit.

16. B

Their mutual gravitational attraction forces F are the same since they are action and reaction pair.

Since both X and Y takes the same time to revolve 1 cycle, they must have the same period, thus the same angular speed ω.

✓ (1) By $a = r\omega^2$, as the radius of X is twice that of Y, the acceleration of X is also twice that of Y.

× (2) By $v = r\omega$, as the radius of X is twice that of Y, the speed of X should also be twice that of Y.

✓ (3) By $F = mr\omega^2$, as the radius of X is twice that of Y, the mass of X is half that of Y.

17. B

The satellite moves round the planet in circular motion, thus the gravitational force provides the centripetal force.

As $\frac{GMm}{r^2} = mr\omega^2$

$$\therefore \frac{GM}{r^3} = \omega^2 = (\frac{2\pi}{T})^2$$

$$\therefore T^2 = \frac{4\pi^2 r^3}{GM} \propto \frac{r^3}{M}$$

$$\therefore (\frac{T_1}{T_2})^2 = (\frac{r_1}{r_2})^3(\frac{M_2}{M_1}) \qquad \therefore (\frac{T_1}{27.3})^2 = (\frac{3}{1})^3(\frac{1}{318})$$

$\therefore T_1 = 8.0$ days

18. D

✓ (1) $\frac{GMm}{r^2} = mr\omega^2$ $\therefore \frac{GM}{r^3} = \omega^2 = (\frac{2\pi}{T})^2$ $\therefore T^2 \propto r^3$ $\therefore r_X > r_Y \Rightarrow T_X > T_Y$

✓ (2) $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\therefore v^2 = \frac{GM}{r}$ $\therefore r_X > r_Y \Rightarrow v_X < v_Y$

✓ (3) $F = \frac{GMm}{r^2} \propto \frac{1}{r^2}$ $\therefore r_X > r_Y \Rightarrow F_X < F_Y$

19. B

$$g = \frac{GM}{r^2} \propto \frac{1}{r^2}$$

$$g = (9.81) \times (\frac{6380}{6380+3200})^2 = 4.35\ \text{N kg}^{-1}$$

20. D

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore v \propto \sqrt{\frac{1}{r}}$$

$$\therefore \frac{v_X}{v_Y} = \sqrt{\frac{r_Y}{r_X}} = \sqrt{\frac{(R+2R)}{(R+R)}} = \sqrt{\frac{3}{2}}$$

21. D

① $s = ut + \frac{1}{2}at^2$ $\therefore (0) = v_1 t + \frac{1}{2}(-g)t^2$ $\therefore g = \frac{2v_1}{t}$

② $mg = \frac{m{v_2}^2}{R}$ $\therefore g = \frac{{v_2}^2}{R}$

Combining the two expressions :

$\therefore \frac{{v_2}^2}{R} = \frac{2v_1}{t}$ $\therefore R = \frac{{v_2}^2 \cdot t}{2v_1}$

22. D

✓ (1) $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\therefore v^2 = \frac{GM}{r}$ $\therefore$ v depends on G

✓ (2) $\frac{GMm}{r^2} = mr\omega^2$ $\therefore \frac{GM}{r^3} = \omega^2 = (\frac{2\pi}{T})^2$ $\therefore T^2 = \frac{4\pi^2 r^3}{GM}$ $\therefore$ T depends on G

✓ (3) $a = g = \frac{GM}{r^2}$ $\therefore$ a depends on G

23. B

Let the mass of the point mass be m.

By $F = \frac{GMm}{r^2}$

For sphere X: $(120) = \frac{G(M)m}{d^2}$

For sphere Y: $F = \frac{G(3M)m}{(2d)^2} = \frac{3}{4} \times \frac{G(M)m}{(d)^2} = \frac{3}{4} \times (120) = 90\ \text{N}$

24. B

× A. In circular motion, there is centripetal acceleration, thus the resultant force cannot be zero.

✓ B. The only force acting on the satellite is the gravitational force (weight), thus it is the resultant force. This force also provides the centripetal force for the circular motion.

× C. Centripetal force itself is the resultant force towards the centre.

× D. For a satellite moving in constant speed, the rockets of the satellite is shut down without exerting force.

25. D

✓ A. $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $\therefore v = \sqrt{\frac{GM}{R}}$

✓ B. The centripetal force must be directing towards the centre, thus along the orbital plane of Q.

✓ C. Acceleration of the satellite is equal to the acceleration due to gravity at that position.

By $a = g = \frac{GM}{r^2}$ Since they have the same radius r, they must have the same acceleration g.

× D. $\frac{GMm}{R^2} = \cancel{m}R\omega^2 = mR\left(\frac{2\pi}{T}\right)^2$ $\therefore$ They have the same period.

26. B

$$g_E = \frac{GM}{R^2} = g$$

$$g_M = \frac{G(\frac{1}{10}M)}{(\frac{1}{2}R)^2} = \frac{4}{10} \times \frac{GM}{R^2}$$

$$\therefore g_M = 0.4\,g_E = 0.4\,g$$

27. B

× A. In circular orbit, the Earth's gravitational force must provide the centripetal force for the circular motion. Thus, the spacecraft and astronaut cannot be so far that the gravitational force is negligible.

✓ B. Since the astronaut and the spacecraft are both moving with the same acceleration due to gravity, their own weight provides their own acceleration, $mg = ma$, thus $a = g$.
Therefore, there is no normal reaction acting on the astronaut, and thus weightlessness is experienced.

× C. At the state of weightlessness, there is no reaction force acting on the astronaut.

× D. The gravitational force is the only force acting on the astronaut.
Centripetal force is the net force towards the centre.
Thus, the gravitational force provides the centripetal force, but not balanced.

28. B

The acceleration g at different position of the Earth is inversely proportional to the square of radius of that position.

$g \propto \frac{1}{r^2}$ $\therefore \frac{g_2}{g_1} = \left(\frac{r_1}{r_2}\right)^2$ $\therefore \frac{g_2}{(g)} = \left(\frac{R}{R+R}\right)^2$ $\therefore g_2 = \frac{1}{4}g$

29. B

These two forces are action and reaction pair, thus, they must be equal in magnitude but opposite in direction.

30. B

As $\dfrac{GMm}{r^2} = mr\omega^2$

$\therefore \dfrac{GM}{r^3} = \omega^2 = (\dfrac{2\pi}{T})^2$

$\therefore \dfrac{(6.67\times10^{-11})(6.0\times10^{24})}{(7.2\times10^6)^3} = (\dfrac{2\pi}{T})^2$

$\therefore T = 6067 \text{ s} = 1.7 \text{ hours}$

31. C

The acceleration due to gravity : $a = g = \dfrac{GM}{r^2}$

Thus, when the object is closer to the planet, r decreases, the acceleration due to gravity g increases.

As the slope of the v - t graph represents acceleration,

greater acceleration means greater slope,

thus, the slope of the graph should gradually increase, as shown in C.

32. C

By $\dfrac{GMm}{r^2} = mr\omega^2$

$\therefore \omega^2 = \dfrac{GM}{r^3}$

$\therefore \omega^2$ depends on r and M only but not depend on m.

The following list of formulae may be found useful :

Centripetal acceleration	$a = \dfrac{v^2}{r} = \omega^2 r$
Newton's law of gravitation	$F = \dfrac{Gm_1m_2}{r^2}$

Use the following data wherever necessary :

Acceleration due to gravity	$g = 9.81 \text{ m s}^{-2}$	(close to the Earth)
Universal gravitational constant	$G = 6.67\times10^{-11} \text{ N m}^2 \text{ kg}^{-2}$	

Part A : HKAL examination questions

1. **< HKAL 1985 Paper IIB - 2 >**

(a) Billy and Lily have heard about the possibility of placing a communications satellite in an orbit such that it remains vertically above the same place on the surface of the Earth. Comment on each of the following statements.

(1) Billy said, "The satellite must be so far away that it is not affected by the Earth's gravity."

(2) Lily said, "There is a communication satellite directly above Hong Kong." (4 marks)

(b) Newton once argued that if a cannon-ball were fired horizontally at high enough speed from any point on the Earth surface, it would eventually return and strike the cannon from behind, by considering the Earth to be a non-rotating sphere.

(i) Take the radius of the Earth to be 6400 km, and assume that the cannon-ball moves close to the Earth's surface, what would be the least time for it to arrive at the cannon again ? (3 marks)

(ii) Give any three reasons with brief explanations that in reality, why the cannon-ball would not arrive at the cannon again. (3 marks)

2. < HKAL 1990 Paper IIB - 7 >

(a) Explain the meaning of the 'parking orbit' for satellites moving around the Earth. (2 marks)

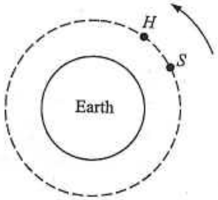
(b) Assume the Earth to be a sphere of uniform density. Calculate the height of a satellite which is in a 'parking orbit', given that the radius of the Earth is 6400 km. (4 marks)

3. < HKAL 1991 Paper IIB - 7 >

A spacecraft has just finished its mission and is returning to the earth. As the spacecraft is moving at a very high speed, air resistance acting on the spacecraft by the atmosphere would cause its surface to reach a very high temperature. State and explain one property of the surface material of the spacecraft for protecting the astronauts inside the spacecraft. (2 marks)

4. < HKAL 1998 Paper I - 1 >

A space shuttle S is sent to service the Hubble Space Telescope H which is in a circular orbit 6.0×10^5 m above the Earth's surface. The space shuttle finally reaches the same orbit as H and its thrust rockets are shut down. Given that the radius of the Earth is 6.4×10^6 m.



(a) What is the apparent weight (feeling of weight) of an astronaut of mass 70 kg inside the shuttle ? (1 mark)

(b) (i) Calculate the value of the gravitational field strength in the orbit. (3 marks)

(ii) Calculate the speed of the space-shuttle S in the orbit. (2 marks)

(iii) Calculate the time take for the space-shuttle S to revolve one revolution in the orbit. (2 marks)

5. < HKAL 2002 Paper I - 6 >

A space shuttle is launched into a circular orbit around the Earth at an altitude of 2.4×10^5 m. Given that the radius of the Earth is 6.4×10^6 m.

(a) Calculate the orbital speed of the shuttle in the orbit. (3 marks)

(b) Calculate the gravitational force acting on an astronaut of mass 80 kg in the shuttle. (2 marks)

6. < HKAL 2005 Paper I - 6 >

(a) A spacecraft is launched with a total initial mass of 4.80×10^5 kg at take-off. The rocket engine propels hot exhaust gas at a constant speed of 2600 m s^{-1} relative to the rocket in a downward direction. Assume that 2300 kg of gas is expelled in the first second. Neglect air resistance.

(i) Calculate the average upward force acting on the rocket due to the exhaust gas during the first second. (2 marks)

(ii) Calculate the acceleration of the rocket during the first second, if the change of mass of the rocket is assumed to be negligible in the first second. (2 marks)

(iii) If the rocket keeps expelling exhaust gas at the same rate for the first 20 s, explain how the acceleration of the rocket will change. (2 marks)

6. (b) The spacecraft of mass 7800 kg then enters a circular orbit around the Earth at a height of 3.43×10^5 m above the surface of the Earth. The radius of the Earth is 6.37×10^6 m.

(i) Calculate the speed of the spacecraft in the orbit. (4 marks)

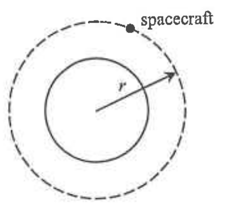
(ii) Calculate the time taken for the spacecraft to orbit around the Earth for 24 times. (2 marks)

(c) Give **TWO** reasons why an aircraft is unable to fly in space where there is no air. (2 marks)

Part B : HKDSE examination questions

7. **< HKDSE Sample Paper IB - 11 >**

A spacecraft of mass 7.80×10^3 kg enters a circular orbit of radius r around the Earth.



(a) Show that the speed of the spacecraft in the orbit is given by $\sqrt{\frac{g}{r}}R_E$ where g is the acceleration due to gravity at the Earth's surface and R_E is the radius of the Earth. (2 marks)

(b) How long does it take for the spacecraft to orbit the Earth 14 times ? (3 marks)

Given : radius of the orbit $r = 6.71 \times 10^6$ m ; radius of the Earth $R_E = 6.37 \times 10^6$ m

8. **< HKDSE Practice Paper IB - 4 >**

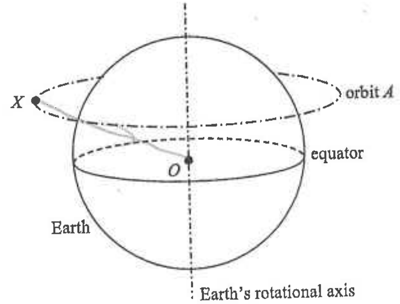
A communication satellite moves in a circular orbit around the Earth with a period of 24 hours and remains above a certain place on the equator.

Given : radius of the Earth $r_E = 6400$ km

(a) (i) Find the orbital radius of the communication satellite. (3 marks)

(ii) Determine the orbital speed of the communication satellite. (2 marks)

(b) In the Figure below, X is a point in space and O is the centre of the Earth.



(i) A satellite is at X. In the above Figure, draw the gravitational force acting on the satellite due to the Earth. (1 mark)

(ii) Briefly explain why the satellites cannot move in a circular orbit A as shown in the Figure under the influence of the Earth's gravitational force only. (1 mark)

HKEAA's Marking Scheme is prepared for the markers' reference. It should not be regarded as a set of model answers. Students and teachers who are not involved in the marking process are advised to interpret the Marking Scheme with care.

Question Solution

1. (a) (1) Billy is wrong,

since the satellite moves in circular orbit, it needs a centripetal force to provide its centripetal acceleration, [1]

so it must be in the Earth's gravitational field so that the gravitational force provides the centripetal force. [1]

(2) Lily is wrong,

since the orbit of a communication satellite must be on the equatorial plane of the Earth, [1]

but Hong Kong is not at the equator, thus no communication satellite is directly above Hong Kong. [1]

(b) (i) $mg = \frac{mv^2}{R}$ [1]

$v = \sqrt{gR} = \sqrt{(9.81)(6.4\times10^6)} = 7924 \text{ m s}^{-1}$

Time : $T = \frac{2\pi R}{v}$ [1]

$= \frac{2\pi\times6.4\times10^6}{7924} = 5070 \text{ s}$ [1]

< OR >

$mg = mR\omega^2 = mR(\frac{2\pi}{T})^2$ [1]

$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^6}{9.81}}$ [1]

$= 5070 \text{ s}$ [1]

(ii) **Reasons** : (Any 3 of the following) [3]

* Earth is not a perfect sphere; g varies over the Earth's surface.
* Earth rotates; when cannon-ball comes back, the cannon may have moved to a different location.
* Air resistance may reduce the speed of the cannon-ball, thus the speed cannot be constant.
* The cannon may hit high buildings (**OR** high mountains), thus it cannot reach the final position.

2. (a) These satellites appear stationary relative to observers on Earth. [2]

(b) $g = \frac{GM}{R^2}$ $\therefore GM = gR^2 = (9.81)(6.4\times10^6)^2 = 4.018\times10^{14}$ [1]

$\frac{GMm}{(R+h)^2} = m(R+h)\omega^2$ $\therefore \frac{GM}{(R+h)^3} = \omega^2 = (\frac{2\pi}{T})^2$ [1]

$\therefore \frac{(4.018\times10^{14})}{(6.4\times10^6+h)^3} = (\frac{2\pi}{24\times3600})^2$ [1]

$\therefore h = 3.60\times10^7 \text{ m}$ < accept $3.58 - 3.62\times10^7$ m > [1]

3. It should have poor thermal conductivity [1]

so that heat will hardly be conducted into spacecraft. [1]

< OR >

It should have high specific heat capacity [1]

so that the rise of temperature is smaller. [1]

< OR >

It should have high melting point [1]

so that the surface of the spacecraft would not melt easily. [1]

4. (a) Feeling of weight = 0 N **OR** He feels weightless. [1]

(b) (i) $g = \frac{GM_E}{r^2} \propto \frac{1}{r^2}$ [1]

$\therefore \frac{g'}{(9.81)} = (\frac{6.4\times10^6}{6.4\times10^6+6\times10^5})^2$ [1]

$\therefore g' = 8.20 \text{ N kg}^{-1}$ [1]

(ii) $mg = \frac{mv^2}{r}$ $\therefore (8.20) = \frac{v^2}{(6.4\times10^6+6\times10^5)}$ [1]

$\therefore v = 7580 \text{ m s}^{-1}$ < accept 7570 to 7600 m s^{-1} > [1]

(iii) $T = \frac{2\pi r}{v} = \frac{2\pi(6.4\times10^6+6\times10^5)}{(7580)}$ [1]

$= 5800 \text{ s}$ < accept 5790 s to 5810 s > [1]

5. (a) $g = \frac{GM}{R^2}$ $\therefore GM = gR^2 = (9.81)(6.4\times10^6)^2 = 4.018\times10^{14}$ [1]

By $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]

$\therefore v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.018\times10^{14}}{(2.4\times10^5+6.4\times10^6)}} = 7780 \text{ m s}^{-1}$ < accept 7800 m s^{-1} > [1]

OR

$g' = (9.81)\times\left(\frac{6.4\times10^6}{6.4\times10^6+2.4\times10^5}\right)^2 = 9.114$ [1]

By $mg' = \frac{mv^2}{r}$ [1]

$\therefore v = \sqrt{g'r} = \sqrt{(9.114)(6.4\times10^6+2.4\times10^5)} = 7780 \text{ m s}^{-1}$ [1]

5. (b) $F = \frac{mv^2}{r} = \frac{(80)(7780)^2}{(2.4\times10^5+6.4\times10^6)}$ [1]

$= 729$ N [1]

OR

$g \propto \frac{1}{r^2}$ $\therefore$ $\frac{g}{(9.81)} = \frac{(6.4\times10^6)^2}{(6.4\times10^6+2.4\times10^5)^2}$ [1]

$\therefore$ $g = 9.11$ N kg^{-1}

$F = mg = (80)\times(9.11) = 729$ N [1]

6. (a) (i) Force on exhaust gas by rocket :

$F = \frac{mv-mu}{t} = \frac{m}{t}(v-u) = (2.30\times10^3)(2600-0)$ [1]

$= 5.98\times10^6$ N [1]

By Newton's 3rd law, force on rocket by exhaust gas is also 5.98×10^6 N

(ii) $F - mg = ma$ [1]

$\therefore$ $(5.98\times10^6) - (4.80\times10^5)(9.81) = (4.80\times10^5)\,a$

$\therefore$ $a = 2.65$ m s^{-2} [1]

(iii) Since the average thrust remains unchanged, as the mass of the rocket gradually decreases, [1]
the acceleration of the rocket would gradually increase. [1]

(b) (i) At the Earth's surface,

$g = \frac{GM}{R^2}$ $\therefore$ $GM = gR^2$ [1]

For circular motion, $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]

$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{(9.81)(6.37\times10^6)^2}{(3.43\times10^5+6.37\times10^6)}}$ [1]

$= 7700$ m s^{-1} [1]

(ii) $t = \frac{2\pi\cdot r}{v}\times24$

$= \frac{2\pi\times(3.43\times10^5+6.37\times10^6)}{7700}\times24$ [1]

$= 1.31\times10^5$ s < accept 1.3×10^5 s > < accept 36.5 h > [1]

(c) ① Aircraft needs air to provide a lift force on its wings. [1]

② Aircraft draws air into the engine for combustion of the fuel. [1]

7. (a) $g = \frac{GM}{R_E^2}$ $\therefore$ $GM = gR_E^2$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]

$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR_E^2}{r}} = \sqrt{\frac{g}{r}}\cdot R_E$ [1]

(b) $v = \sqrt{\frac{gR_E^2}{r}} = \sqrt{\frac{(9.81)(6.37\times10^6)^2}{(6.71\times10^6)}} = 7702$ m s^{-1} [1]

$t = \frac{2\pi\cdot r}{v}\times14 = \frac{2\pi\times(6.71\times10^6)}{7702}\times14$ [1]

$= 7.66\times10^4$ s < accept 7.7×10^4 s > < accept 21.3 hr > [1]

8. (a) (i) $g = \frac{GM}{r_E^2}$ $\therefore$ $GM = g r_E^2 = (9.81)(6.4\times10^6)^2 = 4.018\times10^{14}$ [1]

$\frac{GMm}{r^2} = mr\omega^2$ $\therefore$ $\frac{GM}{r^3} = \omega^2 = (\frac{2\pi}{T})^2$ [1]

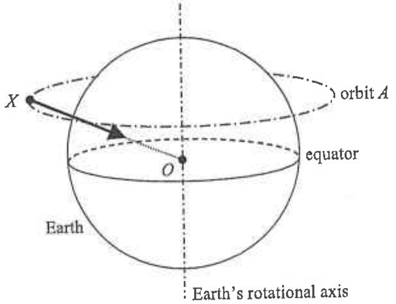
$\therefore$ $\frac{(4.018\times10^{14})}{r^3} = (\frac{2\pi}{24\times3600})^2$

$\therefore$ $r = 4.24\times10^7$ m < accept 4.26×10^7 m > [1]

(ii) $v = \frac{2\pi r}{T}$ [1]

$= \frac{2\pi(4.24\times10^7)}{(24\times3600)} = 3080$ m s^{-1} [1]

(b) (i) [1]



(ii) Any **ONE** of the followings : [1]

* The centre of the orbit is not at the centre of the Earth.
* The direction of the centripetal force is different from the direction of the gravitational force on the satellite.
* The plane of orbit of a satellite must pass through the centre of the Earth.